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Supplementary Document: Real-time Indirect Illumination of Emissive Inhomogeneous Volumes using Layered Polygonal Area Lights

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Appendix A: Derivation of Equation 8

To simplify the exposition, let us introduce an interpolation parameter $t = (s - s_k)/(s_{k+1} - s_k) \in [0, 1]$. By substituting Equations 6 and 7 into Equation 5, we can approximate the original integration as follows:

$$\begin{aligned}\mathcal{I} &= \int_{s_k}^{s_{k+1}} L(s) e^{-\int_{s_k}^{s} \bar{\sigma}_{t,k}(u) du} ds \\ &\approx \int_0^1 ((1-t)L(s_k) + tL(s_{k+1})) e^{-\bar{\sigma}_{t,k}\delta_k t} ds \\ &= (L(s_{k+1}) - L(s_k)) \int_0^1 t e^{-\bar{\sigma}_{t,k}\delta_k t} dt \\ &\quad + L(s_k) \int_0^1 e^{-\bar{\sigma}_{t,k}\delta_k t} dt. \end{aligned} \tag{A.1}$$

From the transformation above, the problem is simplified to solving the integral in each term of the above equation. The first integral can be solved as follows:

$$\begin{aligned}\int_0^1 t e^{-\bar{\sigma}_{t,k}\delta_k t} dt &= -\frac{1}{\bar{\sigma}_{t,k}\delta_k} \left(\left[t e^{-\bar{\sigma}_{t,k}\delta_k t} \right]_{t=0}^{t=1} - \int_0^1 e^{-\bar{\sigma}_{t,k}\delta_k t} dt \right) \\ &= -\frac{1}{\bar{\sigma}_{t,k}\delta_k} \left(e^{-\bar{\sigma}_{t,k}\delta_k} + \frac{e^{-\bar{\sigma}_{t,k}\delta_k}}{\bar{\sigma}_{t,k}\delta_k} - \frac{1}{\bar{\sigma}_{t,k}\delta_k} \right) \\ &= \frac{e^{-\bar{\sigma}_{t,k}\delta_k}}{(\bar{\sigma}_{t,k}\delta_k)^2} \left(-\bar{\sigma}_{t,k}\delta_k + e^{-\bar{\sigma}_{t,k}\delta_k} - 1 \right). \end{aligned}$$

In the same manner, the second integral can be solved as follows:

$$\begin{aligned}\int_0^1 e^{-\bar{\sigma}_{t,k}\delta_k t} dt &= -\frac{1}{\bar{\sigma}_{t,k}\delta_k} \left[e^{-\bar{\sigma}_{t,k}\delta_k t} \right]_{t=0}^{t=1} \\ &= -\frac{1}{\bar{\sigma}_{t,k}\delta_k} \left(e^{-\bar{\sigma}_{t,k}\delta_k t} - 1 \right). \end{aligned}$$

Thus, Equation 8 can be obtained by substituting these calculation results into Equation A.1.

Appendix B: Additional results for self-occlusion and obstacles

There are two figures provided in this section; Figure B.1 shows the rendering results for volumes with different magnitudes of extinction within smokes, and Figure B.2 shows the results for volumes partially occluded by an opaque obstacle.

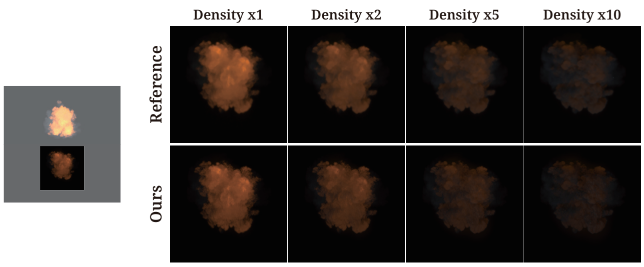

Figure B.1: *Comparison of results rendered with volumes of different magnitudes of extinction within smokes. Regardless of the magnitude of extinction, the results rendered with proposed method are almost identical to those given by physically based path tracing.*

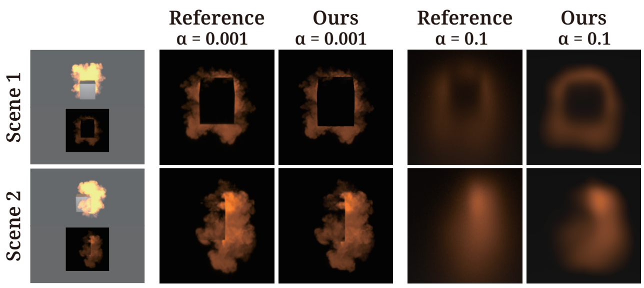

Figure B.2: *Rendering results for volumes containing a cubic opaque obstacle. In the results on each row, the obstacle is located in front or at the side of the volume respectively. As can be seen in these results, the proposed method reproduces the reflected appearances occluded by the obstacle.*